## REVIEW \& SUMMARY

Magnetic Flux The magnetic flux $\Phi_{B}$ through an area $A$ in a magnetic field $\vec{B}$ is defined as

$$
\begin{equation*}
\Phi_{B}=\int \vec{B} \cdot d \vec{A}, \tag{30-1}
\end{equation*}
$$

where the integral is taken over the area. The SI unit of magnetic flux is the weber, where $1 \mathrm{~Wb}=1 \mathrm{~T} \cdot \mathrm{~m}^{2}$. If $\vec{B}$ is perpendicular to the area and uniform over it, Eq. 30-1 becomes

$$
\begin{equation*}
\Phi_{B}=B A \quad(\vec{B} \perp A, \vec{B} \text { uniform }) . \tag{30-2}
\end{equation*}
$$

Faraday's Law of Induction If the magnetic flux $\Phi_{B}$ through an area bounded by a closed conducting loop changes with time, a current and an emf are produced in the loop; this process is called induction. The induced emf is

$$
\begin{equation*}
\mathscr{E}=-\frac{d \Phi_{B}}{d t} \quad \text { (Faraday's law). } \tag{30-4}
\end{equation*}
$$

If the loop is replaced by a closely packed coil of $N$ turns, the induced emf is

$$
\begin{equation*}
\mathscr{E}=-N \frac{d \Phi_{B}}{d t} . \tag{30-5}
\end{equation*}
$$

Lenz's Law An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current. The induced emf has the same direction as the induced current.

Emf and the Induced Electric Field An emf is induced by a changing magnetic flux even if the loop through which the flux is changing is not a physical conductor but an imaginary line. The changing magnetic field induces an electric field $\vec{E}$ at every point of such a loop; the induced emf is related to $\vec{E}$ by

$$
\begin{equation*}
\mathscr{E}=\oint \vec{E} \cdot d \vec{s} \tag{30-19}
\end{equation*}
$$

where the integration is taken around the loop. From Eq. $30-19$ we can write Faraday's law in its most general form,

$$
\begin{equation*}
\oint \vec{E} \cdot d \vec{s}=-\frac{d \Phi_{B}}{d t} \quad \text { (Faraday's law). } \tag{30-20}
\end{equation*}
$$

$A$ changing magnetic field induces an electric field $\vec{E}$.
Inductors An inductor is a device that can be used to produce a known magnetic field in a specified region. If a current $i$ is established through each of the $N$ windings of an inductor, a magnetic flux $\Phi_{B}$ links those windings. The inductance $L$ of the inductor is

$$
\begin{equation*}
L=\frac{N \Phi_{B}}{i} \quad \text { (inductance defined) } \tag{30-28}
\end{equation*}
$$

The SI unit of inductance is the henry $(\mathrm{H})$, where 1 henry $=1 \mathrm{H}=1$ $\mathrm{T} \cdot \mathrm{m}^{2} / \mathrm{A}$. The inductance per unit length near the middle of a long solenoid of cross-sectional area $A$ and $n$ turns per unit length is

$$
\begin{equation*}
\frac{L}{l}=\mu_{0} n^{2} A \quad \text { (solenoid). } \tag{30-31}
\end{equation*}
$$

Self-Induction If a current $i$ in a coil changes with time, an emf is induced in the coil. This self-induced emf is

$$
\begin{equation*}
\mathscr{E}_{L}=-L \frac{d i}{d t} . \tag{30-35}
\end{equation*}
$$

The direction of $\mathscr{E}_{L}$ is found from Lenz's law:The self-induced emf acts to oppose the change that produces it.

Series RL Circuits If a constant emf $\mathscr{E}$ is introduced into a sin-gle-loop circuit containing a resistance $R$ and an inductance $L$, the current rises to an equilibrium value of $\mathscr{E} / R$ according to

$$
\begin{equation*}
i=\frac{\mathscr{E}}{R}\left(1-e^{-t / \tau_{L}}\right) \quad \text { (rise of current). } \tag{30-41}
\end{equation*}
$$

Here $\tau_{L}(=L / R)$ governs the rate of rise of the current and is called the inductive time constant of the circuit. When the source of constant emf is removed, the current decays from a value $i_{0}$ according to

$$
\begin{equation*}
i=i_{0} e^{-t / \tau_{L}} \quad \text { (decay of current). } \tag{30-45}
\end{equation*}
$$

Magnetic Energy If an inductor $L$ carries a current $i$, the inductor's magnetic field stores an energy given by

$$
\begin{equation*}
U_{B}=\frac{1}{2} L i^{2} \quad \text { (magnetic energy). } \tag{30-49}
\end{equation*}
$$

If $B$ is the magnitude of a magnetic field at any point (in an inductor or anywhere else), the density of stored magnetic energy at that point is

$$
\begin{equation*}
u_{B}=\frac{B^{2}}{2 \mu_{0}} \quad \text { (magnetic energy density). } \tag{30-55}
\end{equation*}
$$

Mutual Induction If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$
\begin{equation*}
\mathscr{E}_{2}=-M \frac{d i_{1}}{d t} \tag{30-64}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathscr{E}_{1}=-M \frac{d i_{2}}{d t} \tag{30-65}
\end{equation*}
$$

where $M$ (measured in henries) is the mutual inductance.

## Q U ESTIONS

1 If the circular conductor in Fig. 30-21 undergoes thermal expansion while it is in a uniform magnetic field, a current is induced


Fig. 30-21 Question 1.
clockwise around it. Is the magnetic field directed into or out of the page?
2 The wire loop in Fig. 30-22a is subjected, in turn, to six uniform magnetic fields, each directed parallel to the $z$ axis, which is directed out of the plane of the figure. Figure $30-22 b$ gives the $z$ components $B_{z}$ of the fields versus time $t$. (Plots 1 and 3 are parallel; so are plots 4 and 6 . Plots 2 and 5 are parallel to the time axis.) Rank the six plots according to the emf induced in
the loop, greatest clockwise emf first, greatest counterclockwise emf last.
(a)


Fig. 30-22 Question 2.
3 In Fig. 30-23, a long straight wire with current $i$ passes (without touching) three rectangular wire loops with edge lengths $L, 1.5 L$, and $2 L$. The loops are widely spaced (so as not to affect one another). Loops 1 and 3 are symmetric about the long wire. Rank the loops according to the size of the current induced in them if current $i$ is (a) constant and (b) increasing, greatest first.


Fig. 30-23 Question 3.
4 Figure 30-24 shows two circuits in which a conducting bar is slid at the same speed $v$ through the same uniform magnetic field and along a U-shaped wire. The parallel lengths of the wire are separated by $2 L$ in circuit 1 and by $L$ in circuit 2 . The current induced in circuit 1 is counterclockwise. (a) Is the magnetic field into or out of the page? (b) Is the current induced in circuit 2 clockwise or counterclockwise? (c) Is the emf induced in circuit 1 larger than, smaller than, or the same as that in circuit 2?


Fig. 30-24 Question 4.
5 Figure 30-25 shows a circular region in which a decreasing uniform magnetic field is directed out of the page, as well as four concentric circular paths. Rank the paths according to the magnitude of $\oint \vec{E} \cdot d \vec{s}$ evalu-


Fig. 30-25 Question 5.

6 In Fig. 30-26, a wire loop has been bent so that it has three segments: segment $b c$ (a quarter-circle), $a c$ (a square corner), and $a b$ (straight). Here are three choices for a magnetic field through the loop:
(1) $\vec{B}_{1}=3 \hat{\mathrm{i}}+7 \hat{\mathrm{j}}-5 t \hat{\mathrm{k}}$,
(2) $\vec{B}_{2}=5 t \hat{\mathrm{i}}-4 \hat{\mathrm{j}}-15 \hat{\mathrm{k}}$,
(3) $\vec{B}_{3}=2 \hat{\mathrm{i}}-5 t \hat{\mathrm{j}}-12 \hat{\mathrm{k}}$,
where $\vec{B}$ is in milliteslas and $t$ is in seconds. Without written calculation, rank the choices according to (a) the work done per unit charge in setting up the induced current and (b) that induced current, greatest first. (c) For each choice, what is the direction of the induced current in the figure?

7 Figure 30-27 shows a circuit with two


Fig. 30-26
Question 6. identical resistors and an ideal inductor. Is the current through the central resistor more than, less than, or the same as that through the other resistor (a) just after the closing of switch $S$, (b) a long time after that, (c) just after $S$ is reopened a long time later, and (d) a long time after that?


Fig. 30-27 Question 7.
8 The switch in the circuit of Fig. 30-15 has been closed on $a$ for a very long time when it is then thrown to $b$. The resulting current through the inductor is indicated in Fig. 30-28 for four sets of values for the resistance $R$ and inductance $L$ : (1) $R_{0}$ and $L_{0}$, (2) $2 R_{0}$ and $L_{0}$, (3) $R_{0}$ and $2 L_{0}$, (4) $2 R_{0}$ and $2 L_{0}$. Which set goes with which curve?
9 Figure 30-29 shows three circuits with identical batteries, inductors,


Fig. 30-28 Question 8. and resistors. Rank the circuits, greatest first, according to the current through the resistor labeled $R$ (a) long after the switch is closed, (b) just after the switch is reopened a long time later, and (c) long after it is reopened.


Fig. 30-29 Question 9 .
10 Figure 30-30 gives the variation with time of the potential difference $V_{R}$ across a resistor in three circuits wired as shown in Fig. 30-16. The circuits contain the same resistance $R$ and emf $\mathscr{E}$ but differ in the inductance $L$. Rank the circuits according to the value of $L$, greatest first.


Fig. 30-30 Question 10 .

-2 A certain elastic conducting material is stretched into a circular loop of 12.0 cm radius. It is placed with its plane perpendicular to a uniform 0.800 T magnetic field. When released, the radius of the loop starts to shrink at an instantaneous rate of $75.0 \mathrm{~cm} / \mathrm{s}$. What emf is induced in the loop at that instant?
-3 ssm www In Fig. 30-32, a 120-turn coil of radius 1.8 cm and resistance $5.3 \Omega$ is coaxial with a solenoid of 220 turns $/ \mathrm{cm}$ and diameter 3.2 cm . The solenoid current drops from 1.5 A to zero in time interval $\Delta t=25 \mathrm{~ms}$. What current is induced in the coil during $\Delta t$ ?
-4 A wire loop of radius 12 cm and resistance $8.5 \Omega$ is located in a uniform magnetic field $\vec{B}$ that changes in magnitude as given in Fig. 30-33. The vertical axis scale is set by $B_{s}=$ 0.50 T , and the horizontal axis scale


Fig. 30-32 Problem 3.

$t(\mathrm{~s})$
Fig. 30-33 Problem 4. is set by $t_{s}=6.00 \mathrm{~s}$. The loop's plane is perpendicular to $\vec{B}$. What emf is induced in the loop during time intervals (a) 0 to 2.0 s , (b) 2.0 s to 4.0 s , and (c) 4.0 s to 6.0 s ?
${ }^{-5}$ In Fig. 30-34, a wire forms a closed circular loop, of radius $R=$ 2.0 m and resistance $4.0 \Omega$. The circle is centered on a long straight wire; at time $t=0$, the current in the long straight wire is 5.0 A rightward. Thereafter, the current


Fig. 30-34 Problem 5. changes according to $i=5.0 \mathrm{~A}-$ $\left(2.0 \mathrm{~A} / \mathrm{s}^{2}\right) t^{2}$. (The straight wire is insulated; so there is no electrical contact between it and the wire of the loop.) What is the magnitude of the current induced in the loop at times $t>0$ ?
-6 Figure 30-35a shows a circuit consisting of an ideal battery
(a)


(b)
$t$ (s)

Fig. 30-35 Problem 6.
with emf $\mathscr{E}=6.00 \mu \mathrm{~V}$, a resistance $R$, and a small wire loop of area $5.0 \mathrm{~cm}^{2}$. For the time interval $t=10 \mathrm{~s}$ to $t=20 \mathrm{~s}$, an external magnetic field is set up throughout the loop. The field is uniform, its direction is into the page in Fig. 30-35a, and the field magnitude is given by $B=a t$, where $B$ is in teslas, $a$ is a constant, and $t$ is in seconds. Figure $30-35 b$ gives the current $i$ in the circuit before, during, and after the external field is set up. The vertical axis scale is set by $i_{s}=2.0 \mathrm{~mA}$. Find the constant $a$ in the equation for the field magnitude.
${ }^{-7}$ In Fig. 30-36, the magnetic flux through the loop increases according to the relation $\Phi_{B}=6.0 t^{2}+7.0 t$, where $\Phi_{B}$ is in milliwebers and $t$ is in seconds. (a) What is the magnitude of the emf induced in the loop when $t=2.0 \mathrm{~s}$ ? (b) Is the direction of the current through $R$ to the right or left?
-8 A uniform magnetic field $\vec{B}$ is perpendicular to the plane of a circular


Fig. 30-36 Problem 7. loop of diameter 10 cm formed from wire of diameter 2.5 mm and resistivity $1.69 \times 10^{-8} \Omega \cdot \mathrm{~m}$. At what rate must the magnitude of $\vec{B}$ change to induce a 10 A current in the loop?
-9 A small loop of area $6.8 \mathrm{~mm}^{2}$ is placed inside a long solenoid that has 854 turns $/ \mathrm{cm}$ and carries a sinusoidally varying current $i$ of amplitude 1.28 A and angular frequency $212 \mathrm{rad} / \mathrm{s}$. The central axes of the loop and solenoid coincide. What is the amplitude of the emf induced in the loop?

- 10 Figure 30-37 shows a closed loop of wire that consists of a pair of equal semicircles, of radius 3.7 cm , lying in mutually perpendicular planes. The loop was formed by folding a flat circular loop along a diameter until the two halves became perpendicular to each other. A uniform magnetic field $\vec{B}$ of magnitude 76 mT is directed perpendicular to the fold diameter and makes equal angles (of $45^{\circ}$ ) with the planes of the semicircles. The magnetic field is reduced to zero at a uniform rate during a time interval of 4.5 ms . During this interval, what are the (a) magnitude and (b) direction (clockwise or counterclockwise when viewed along the direction of $\vec{B}$ ) of the emf induced in the loop?


Fig. 30-37 Problem 10.
-•11 A rectangular coil of $N$ turns and of length $a$ and width $b$ is rotated at frequency $f$ in a uniform magnetic field $\vec{B}$, as indicated in Fig. 30-38. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact. (a) Show that the emf induced in the coil is given (as a function of time $t$ ) by

$$
\mathscr{E}=2 \pi f N a b B \sin (2 \pi f t)=\mathscr{E}_{0} \sin (2 \pi f t) .
$$

This is the principle of the commercial alternating-current generator. (b) What value of $N a b$ gives an emf with $\mathscr{E}_{0}=150 \mathrm{~V}$ when the loop is rotated at $60.0 \mathrm{rev} / \mathrm{s}$ in a uniform magnetic field of 0.500 T ?


Fig. 30-38 Problem 11.
-•12 In Fig. 30-39, a wire loop of lengths $L=$ 40.0 cm and $W=25.0 \mathrm{~cm}$ lies in a magnetic field $\vec{B}$. What are the (a) magnitude $\mathscr{E}$ and (b) direction (clockwise or counterclockwise-or "none" if $\mathscr{E}=0$ ) of the emf induced in the loop if $\vec{B}=\left(4.00 \times 10^{-2} \mathrm{~T} / \mathrm{m}\right) y \hat{\mathrm{k}}$ ? What are (c) $\mathscr{E}$ and (d) the direction if $\vec{B}=\left(6.00 \times 10^{-2} \mathrm{~T} / \mathrm{s}\right) t \hat{\mathrm{k}}$ ? What are (e) $\mathscr{E}$ and (f) the direction if $\vec{B}=\left(8.00 \times 10^{-2} \mathrm{~T} / \mathrm{m} \cdot \mathrm{s}\right) y t \hat{\mathrm{k}}$ ? What are (g) $\mathscr{E}$ and (h) the direction if $\vec{B}=\left(3.00 \times 10^{-2} \mathrm{~T} / \mathrm{m} \cdot \mathrm{s}\right) x t \hat{j}$ ? What are (i) $\mathscr{E}$ and (j) the direction if $\vec{B}=\left(5.00 \times 10^{-2} \mathrm{~T} / \mathrm{m} \cdot \mathrm{s}\right) y t \hat{i}$ ?
-•13 ILw One hundred turns of (insulated) copper wire are wrapped around a wooden cylindrical core of cross-sectional area $1.20 \times 10^{-3} \mathrm{~m}^{2}$. The two ends of the wire are connected to a resistor. The total resistance in the circuit is $13.0 \Omega$. If an externally applied uniform longitudinal magnetic field in the core changes from 1.60 T in one direction to 1.60 T in the opposite direction, how much charge flows through a point in the circuit during the change?
-14 In Fig. 30-40a, a uniform magnetic field $\vec{B}$ increases in magnitude with time $t$ as given by Fig. 30-40b, where the vertical axis scale is set by $B_{s}=9.0 \mathrm{mT}$ and the horizontal scale is set by $t_{s}=3.0 \mathrm{~s}$. A circular conducting loop of area $8.0 \times 10^{-4}$ $\mathrm{m}^{2}$ lies in the field, in the plane of the page. The amount of charge $q$ passing point $A$ on the loop is given in Fig. 30-40c as a function of $t$, with the vertical axis scale set by $q_{s}=6.0 \mathrm{mC}$ and the horizontal axis scale again set by $t_{s}=3.0 \mathrm{~s}$. What is the loop's resistance?


Fig. 30-40 Problem 14.
-•15 © A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in Fig. 30-41. The loop contains an ideal battery with emf $\mathscr{E}=$ 20.0 V . If the magnitude of the field varies with time according to $B=0.0420-0.870 t$, with $B$ in teslas and $t$ in seconds, what are (a) the net emf in the circuit and (b) the direction of the (net) current


Fig. 30-41 Problem 15. around the loop?
-•16 ©0 Figure $30-42 a$ shows a wire that forms a rectangle ( $W=20 \mathrm{~cm}, H=30 \mathrm{~cm}$ ) and has a resistance of $5.0 \mathrm{~m} \Omega$. Its interior is split into three equal areas, with magnetic fields $\vec{B}_{1}, \vec{B}_{2}$, and $\vec{B}_{3}$. The fields are uniform within each region and directly out of or into the page as indicated. Figure $30-42 b$ gives the change in the $z$ components $B_{z}$ of the three fields with time $t$; the vertical axis scale is set by $B_{s}=4.0 \mu \mathrm{~T}$ and $B_{b}=-2.5 B_{s}$, and the horizontal axis scale is set by $t_{s}=2.0 \mathrm{~s}$. What are the (a) magnitude and (b) direction of the current induced in the wire?


Fig. 30-42 Problem 16.
-०17 A small circular loop of area $2.00 \mathrm{~cm}^{2}$ is placed in the plane of, and concentric with, a large circular loop of radius 1.00 m . The current in the large loop is changed at a constant rate from 200 A to -200 A (a change in direction) in a time of 1.00 s , starting at $t=0$. What is the magnitude of the magnetic field $\vec{B}$ at the center of the small loop due to the current in the large loop at (a) $t=0$, (b) $t=0.500 \mathrm{~s}$, and (c) $t=1.00 \mathrm{~s}$ ? (d) From $t=0$ to $t=1.00 \mathrm{~s}$, is $\vec{B}$ reversed? Because the inner loop is small, assume $\vec{B}$ is uniform over its area. (e) What emf is induced in the small loop at $t=0.500 \mathrm{~s}$ ?
-•18 In Fig. 30-43, two straight conducting rails form a right angle. A conducting bar in contact with the rails starts at the vertex at time $t=0$ and moves with a constant velocity of $5.20 \mathrm{~m} / \mathrm{s}$ along them. A magnetic field with $B=0.350 \mathrm{~T}$ is directed out of the


Fig. 30-43 Problem 18. page. Calculate (a) the flux through the triangle formed by the rails and bar at $t=3.00 \mathrm{~s}$ and (b) the emf around the triangle at that time. (c) If the emf is $\mathscr{E}=a t^{n}$, where $a$ and $n$ are constants, what is the value of $n$ ?
-•19 ILW An electric generator contains a coil of 100 turns of wire, each forming a rectangular loop 50.0 cm by 30.0 cm . The coil
is placed entirely in a uniform magnetic field with magnitude $B=$ 3.50 T and with $\vec{B}$ initially perpendicular to the coil's plane. What is the maximum value of the emf produced when the coil is spun at $1000 \mathrm{rev} / \mathrm{min}$ about an axis perpendicular to $\vec{B}$ ?
$\bullet 20$ At a certain place, Earth's magnetic field has magnitude $B=0.590$ gauss and is inclined downward at an angle of $70.0^{\circ}$ to the horizontal. A flat horizontal circular coil of wire with a radius of 10.0 cm has 1000 turns and a total resistance of $85.0 \Omega$. It is connected in series to a meter with $140 \Omega$ resistance. The coil is flipped through a half-revolution about a diameter, so that it is again horizontal. How much charge flows through the meter during the flip?
-221 In Fig. 30-44, a stiff wire bent into a semicircle of radius $a=2.0 \mathrm{~cm}$ is rotated at constant angular speed 40 $\mathrm{rev} / \mathrm{s}$ in a uniform 20 mT magnetic field. What are the (a) frequency and (b) amplitude of the emf induced in the loop?
-22 A rectangular loop ( area $=$ $0.15 \mathrm{~m}^{2}$ ) turns in a uniform mag-


Fig. 30-44 Problem 21. netic field, $B=0.20 \mathrm{~T}$. When the angle between the field and the normal to the plane of the loop is $\pi / 2$ rad and increasing at $0.60 \mathrm{rad} / \mathrm{s}$, what emf is induced in the loop?
-223 Ssm Figure 30-45 shows two parallel loops of wire having a common axis. The smaller loop (radius $r$ ) is above the larger loop (radius $R$ ) by a distance $x \gg R$. Consequently, the magnetic field due to the counterclockwise current $i$ in the larger loop is nearly uniform throughout the smaller loop. Suppose that $x$ is increasing at the constant rate $d x / d t=v$. (a) Find an expression for the magnetic flux through the area of the smaller loop as a function of $x$. (Hint: See Eq. 29-27.) In the smaller loop, find (b) an expression for the induced emf and (c) the direction of the induced current.
$\bullet 24$ A wire is bent into three circular segments, each of radius $r=$ 10 cm , as shown in Fig. 30-46. Each segment is a quadrant of a circle, $a b$ lying in the $x y$ plane, $b c$ lying in the $y z$ plane, and $c a$ lying in the $z x$ plane. (a) If a uniform magnetic field $\vec{B}$ points in the positive $x$ direction, what is the magnitude of the emf developed in the wire when $B$ increases at the rate of $3.0 \mathrm{mT} / \mathrm{s}$ ? (b) What is the direction of the cur-


Fig. 30-46 Problem 24. rent in segment $b c$ ?
00025 ©o Two long, parallel copper wires of diameter 2.5 mm carry currents of 10 A in opposite directions. (a) Assuming that their central axes are 20 mm apart, calculate the magnetic flux per meter of wire that exists in the space between those axes. (b) What percentage of this flux lies inside the wires? (c) Repeat part (a) for parallel currents.
-0026 For the wire arrangement in Fig. 30-47, $a=12.0 \mathrm{~cm}$ and $b=$ 16.0 cm . The current in the long straight wire is $i=4.50 t^{2}-10.0 t$, where $i$ is in amperes and $t$ is in seconds. (a) Find the emf in the square loop at $t=3.00 \mathrm{~s}$. (b) What is the direction of the induced current in the loop?


Fig. 30-47 Problem 26.
-0027 ILW As seen in Fig. 30-48, a square loop of wire has sides of length 2.0 cm . A magnetic field is directed out of the page; its magnitude is given by $B=4.0 t^{2} y$, where $B$ is in teslas, $t$ is in seconds, and $y$ is in meters. At $t=2.5 \mathrm{~s}$, what are the (a) magnitude and (b) direction of the emf induced in the loop?


Fig. 30-48 Problem 27.
-0028 In Fig. 30-49, a rectangular loop of wire with length $a=2.2 \mathrm{~cm}$, width $b=0.80 \mathrm{~cm}$, and resistance $R=0.40 \mathrm{~m} \Omega$ is placed near an infinitely long wire carrying current $i=4.7 \mathrm{~A}$. The loop is then moved away from the wire at constant speed $v=3.2 \mathrm{~mm} / \mathrm{s}$. When the center of the loop is at distance $r=1.5 b$, what are (a) the magnitude of the magnetic flux through the loop and (b) the current induced in the loop?

## sec. 30-5 Induction and Energy Transfers

-29 In Fig. 30-50, a metal rod is forced to move with constant velocity $\vec{v}$ along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude $B=0.350 \mathrm{~T}$ points out of the page. (a) If the rails are separated by $L=25.0 \mathrm{~cm}$ and the speed of the rod is $55.0 \mathrm{~cm} / \mathrm{s}$, what emf is generated? (b) If the rod has a resistance of $18.0 \Omega$ and the rails and connector have


Fig. 30-50 Problems 29 and 35.
negligible resistance, what is the current in the rod? (c) At what rate is energy being transferred to thermal energy?
-30 In Fig. 30-51a, a circular loop of wire is concentric with a solenoid and lies in a plane perpendicular to the solenoid's central axis. The loop has radius 6.00 cm . The solenoid has radius 2.00 cm , consists of 8000 turns $/ \mathrm{m}$, and has a current $i_{\text {sol }}$ varying with time $t$ as given in Fig. 30-51b, where the vertical axis scale is set by $i_{s}=1.00$ A and the horizontal axis scale is set by $t_{s}=2.0 \mathrm{~s}$. Figure $30-51 \mathrm{c}$ shows, as a function of time, the energy $E_{\mathrm{th}}$ that is transferred to thermal energy of the loop; the vertical axis scale is set by $E_{s}=$ 100.0 nJ . What is the loop's resistance?


Fig. 30-51 Problem 30.
-31 SSM ILW If 50.0 cm of copper wire (diameter $=1.00 \mathrm{~mm}$ ) is formed into a circular loop and placed perpendicular to a uniform magnetic field that is increasing at the constant rate of 10.0 $\mathrm{mT} / \mathrm{s}$, at what rate is thermal energy generated in the loop?
-32 A loop antenna of area $2.00 \mathrm{~cm}^{2}$ and resistance $5.21 \mu \Omega$ is perpendicular to a uniform magnetic field of magnitude $17.0 \mu \mathrm{~T}$. The field magnitude drops to zero in 2.96 ms . How much thermal energy is produced in the loop by the change in field?
-•33 Figure 30-52 shows a rod of length $L=10.0 \mathrm{~cm}$ that is forced to move at constant speed $v=5.00$ $\mathrm{m} / \mathrm{s}$ along horizontal rails. The rod, rails, and connecting strip at the right form a conducting loop. The rod has resistance $0.400 \Omega$; the rest of the loop has negligible resistance. A current $i=100$ A through the long straight wire at distance $a=10.0 \mathrm{~mm}$ from the loop sets up a (nonuniform) magnetic field through the loop. Find the (a) emf and (b) current induced in the loop. (c) At what rate is thermal energy generated in the rod? (d) What is the magnitude of the force that must be applied to the rod to make it move at constant speed? (e) At what rate does this force do work on the rod?
-•34 In Fig. 30-53, a long rectangular conducting loop, of width $L$, resistance $R$, and mass $m$, is hung in a horizontal, uniform magnetic field $\vec{B}$ that is directed into the page and that exists only above line $a a$. The loop is then dropped; during its fall, it accelerates until it reaches a certain terminal speed $v_{t}$. Ignoring air drag, find an expression for $v_{t}$


Fig. 30-53 Problem 34.
-•35 The conducting rod shown in Fig. 30-50 has length $L$ and is being pulled along horizontal, frictionless conducting rails at a constant velocity $\vec{v}$. The rails are connected at one end with a metal strip. A uniform magnetic field $\vec{B}$, directed out of the page, fills the region in which the rod moves. Assume that $L=10 \mathrm{~cm}$, $v=5.0 \mathrm{~m} / \mathrm{s}$, and $B=1.2 \mathrm{~T}$. What are the (a) magnitude and (b) direction (up or down the page) of the emf induced in the rod? What are the (c) size and (d) direction of the current in the conducting loop? Assume that the resistance of the rod is $0.40 \Omega$ and that the resistance of the rails and metal strip is negligibly small. (e) At what rate is thermal energy being generated in the rod? (f) What external force on the rod is needed to maintain $\vec{v}$ ? (g) At what rate does this force do work on the rod?

## sec. 30-6 Induced Electric Fields

-36 Figure 30-54 shows two circular regions $R_{1}$ and $R_{2}$ with radii $r_{1}=20.0 \mathrm{~cm}$ and $r_{2}=30.0 \mathrm{~cm}$. In $R_{1}$ there is a uniform magnetic field of magnitude $B_{1}=50.0 \mathrm{mT}$ directed into the page, and in $R_{2}$ there is a uniform magnetic field of magnitude $B_{2}=75.0 \mathrm{mT}$ directed out of the page (ignore fringing). Both fields are decreasing at the rate of $8.50 \mathrm{mT} / \mathrm{s}$. Calculate $\oint \vec{E} \cdot d \vec{s}$ for (a) path 1 , (b) path 2 , and (c) path 3 .


Fig. 30-54 Problem 36.
-37 SSM ILW A long solenoid has a diameter of 12.0 cm . When a current $i$ exists in its windings, a uniform magnetic field of magnitude $B=30.0 \mathrm{mT}$ is produced in its interior. By decreasing $i$, the field is caused to decrease at the rate of $6.50 \mathrm{mT} / \mathrm{s}$. Calculate the magnitude of the induced electric field (a) 2.20 cm and (b) 8.20 cm from the axis of the solenoid.

- 38 so A circular region in an $x y$ plane is penetrated by a uniform magnetic field in the positive direction of the $z$ axis. The field's magnitude $B$ (in teslas) increases with time $t$ (in seconds) according to $B=a t$, where $a$ is a constant. The magnitude $E$ of the electric field set up by that in-


Fig. 30-55 Problem 38. crease in the magnetic field is given by Fig. 30-55 versus radial distance $r$; the vertical axis scale is set by $E_{s}=300 \mu \mathrm{~N} / \mathrm{C}$, and the horizontal axis scale is set by $r_{s}=4.00 \mathrm{~cm}$. Find $a$.
-•39 The magnetic field of a cylindrical magnet that has a pole-face diameter of 3.3 cm can be varied sinusoidally between 29.6 T and 30.0 T at a frequency of 15 Hz . (The current in a wire wrapped around a permanent magnet is varied to give this variation in the net field.) At a radial distance of 1.6 cm , what is the amplitude of the electric field induced by the variation?

## sec. 30-7 Inductors and Inductance

-40 The inductance of a closely packed coil of 400 turns is 8.0 mH . Calculate the magnetic flux through the coil when the current is 5.0 mA .
-41 A circular coil has a 10.0 cm radius and consists of 30.0 closely wound turns of wire. An externally produced magnetic field of magnitude 2.60 mT is perpendicular to the coil. (a) If no current is in the coil, what magnetic flux links its turns? (b) When the current in the coil is 3.80 A in a certain direction, the net flux through the coil is found to vanish. What is the inductance of the coil?

- 42 Figure $30-56$ shows a copper strip of width $W=16.0 \mathrm{~cm}$ that has been bent to form a shape that consists of a tube of radius $R=1.8 \mathrm{~cm}$ plus two parallel flat extensions. Current $i=35 \mathrm{~mA}$ is distributed uniformly across the width so that the tube is effectively a one-turn solenoid. Assume that the magnetic field outside the tube is negligible and the field inside the tube is uniform. What are (a) the magnetic field magnitude inside the tube and (b) the inductance of the tube (excluding the flat extensions)?
$\because 43$ (60 Two identical long wires of radius $a=1.53 \mathrm{~mm}$ are parallel and carry identical


Fig. 30-56
Problem 42. currents in opposite directions. Their center-to-center separation is $d=14.2 \mathrm{~cm}$. Neglect the flux within the wires but consider the flux in the region between the wires. What is the inductance per unit length of the wires?

## sec. 30-8 Self-Induction

-44 A 12 H inductor carries a current of 2.0 A . At what rate must the current be changed to produce a 60 V emf in the inductor?

045 At a given instant the current and self-induced emf in an inductor are directed as indicated in Fig. 30-57.
(a) Is the current increasing or decreasing? (b) The induced emf is 17 V ,


Fig. 30-57 Problem 45. and the rate of change of the current is $25 \mathrm{kA} / \mathrm{s}$; find the inductance.
©046 The current $i$ through a 4.6 H inductor varies with time $t$ as shown by the graph of Fig. 30-58, where the vertical axis scale is set by $i_{s}=8.0 \mathrm{~A}$ and the horizontal axis scale is set by $t_{s}=6.0 \mathrm{~ms}$. The inductor has a resistance of $12 \Omega$. Find the magnitude of the induced emf $\mathscr{E}$ during time intervals (a) 0 to 2 ms , (b) 2 ms to 5 ms , and (c) 5 ms to 6 ms . (Ignore the behavior at the ends of the intervals.)


Fig. 30-58 Problem 46.
-047 Inductors in series. Two inductors $L_{1}$ and $L_{2}$ are connected in series and are separated by a large distance so that the magnetic
field of one cannot affect the other. (a) Show that the equivalent inductance is given by

$$
L_{\mathrm{eq}}=L_{1}+L_{2} .
$$

(Hint: Review the derivations for resistors in series and capacitors in series. Which is similar here?) (b) What is the generalization of (a) for $N$ inductors in series?
©048 Inductors in parallel. Two inductors $L_{1}$ and $L_{2}$ are connected in parallel and separated by a large distance so that the magnetic field of one cannot affect the other. (a) Show that the equivalent inductance is given by

$$
\frac{1}{L_{\mathrm{eq}}}=\frac{1}{L_{1}}+\frac{1}{L_{2}} .
$$

(Hint: Review the derivations for resistors in parallel and capacitors in parallel. Which is similar here?) (b) What is the generalization of (a) for $N$ inductors in parallel?
-•49 The inductor arrangement of Fig. 30-59, with $L_{1}=30.0 \mathrm{mH}, L_{2}=$ $50.0 \mathrm{mH}, L_{3}=20.0 \mathrm{mH}$, and $L_{4}=$ 15.0 mH , is to be connected to a varying current source. What is the equivalent inductance of the arrangement? (First see Problems 47 and 48.)


Fig. 30-59 Problem 49.

## sec. 30-9 RL Circuits

- 50 The current in an $R L$ circuit builds up to one-third of its steady-state value in 5.00 s . Find the inductive time constant.
-51 ILW The current in an $R L$ circuit drops from 1.0 A to 10 mA in the first second following removal of the battery from the circuit. If $L$ is 10 H , find the resistance $R$ in the circuit.
$\bullet 52$ The switch in Fig. 30-15 is closed on $a$ at time $t=0$. What is the ratio $\mathscr{E}_{L} / \mathscr{E}$ of the inductor's self-induced emf to the battery's emf (a) just after $t=0$ and (b) at $t=2.00 \tau_{L}$ ? (c) At what multiple of $\tau_{L}$ will $\mathscr{E}_{L} / \mathscr{E}=0.500$ ?
-53 SSM A solenoid having an inductance of $6.30 \mu \mathrm{H}$ is connected in series with a $1.20 \mathrm{k} \Omega$ resistor. (a) If a 14.0 V battery is connected across the pair, how long will it take for the current through the resistor to reach $80.0 \%$ of its final value? (b) What is the current through the resistor at time $t=1.0 \tau_{L}$ ?
-54 In Fig. 30-60, $\mathscr{E}=100 \mathrm{~V}, R_{1}=10.0 \Omega, R_{2}=20.0 \Omega, R_{3}=30.0$ $\Omega$, and $L=2.00 \mathrm{H}$. Immediately after switch S is closed, what are (a) $i_{1}$ and (b) $i_{2}$ ? (Let currents in the indicated directions have positive values and currents in the opposite directions have negative values.) A long time later, what are (c) $i_{1}$ and (d) $i_{2}$ ? The switch is then reopened. Just then, what are (e) $i_{1}$ and (f) $i_{2}$ ? A long time later, what are $(\mathrm{g}) i_{1}$ and (h) $i_{2}$ ?


Fig. 30-60 Problem 54.
-55 SSM A battery is connected to a series $R L$ circuit at time $t=0$. At what multiple of $\tau_{L}$ will the current be $0.100 \%$ less than its equilibrium value?
-56 In Fig. 30-61, the inductor has 25 turns and the ideal battery has an emf of 16 V . Figure 30-62 gives the magnetic flux $\Phi$ through each turn versus the current $i$ through the inductor. The vertical axis scale is set by $\Phi_{s}=4.0 \times 10^{-4} \mathrm{~T} \cdot \mathrm{~m}^{2}$, and the horizontal axis scale is set by $i_{s}=$ 2.00 A. If switch S is closed at time $t=0$, at what rate $d i / d t$ will the current be changing at $t=1.5 \tau_{L}$ ?


Fig. 30-61
Problems 56, 80, 83, and 93.


Fig. 30-62 Problem 56.
-057 © In Fig. 30-63, $R=15 \Omega$, $L=5.0 \mathrm{H}$, the ideal battery has $\mathscr{E}=$ 10 V , and the fuse in the upper branch is an ideal 3.0 A fuse. It has zero resistance as long as the current through it remains less than 3.0 A. If the current reaches 3.0 A , the fuse "blows" and thereafter has infinite resistance. Switch S is closed


Fig. 30-63 Problem 57. at time $t=0$. (a) When does the fuse blow? (Hint: Equation 30-41 does not apply. Rethink Eq. 30-39.) (b) Sketch a graph of the current $i$ through the inductor as a function of time. Mark the time at which the fuse blows.
$\because 58$ ©o Suppose the emf of the battery in the circuit shown in Fig. 30-16 varies with time $t$ so that the current is given by $i(t)=$ $3.0+5.0 t$, where $i$ is in amperes and $t$ is in seconds. Take $R=4.0 \Omega$ and $L=6.0 \mathrm{H}$, and find an expression for the battery emf as a function of $t$. (Hint: Apply the loop rule.)
$\cdots 059$ ssm www In Fig. 30-64, after switch S is closed at time $t=0$, the emf of the source is automatically adjusted to maintain a constant current $i$ through S. (a) Find the current through the inductor as a function of time. (b) At what time is the current through the resistor equal to the current through the inductor?


Fig. 30-64 Problem 59.
$\bullet 060$ A wooden toroidal core with a square cross section has an inner radius of 10 cm and an outer radius of 12 cm . It is wound with one layer of wire (of diameter 1.0 mm and resistance per meter $0.020 \Omega / \mathrm{m}$ ). What are (a) the inductance and (b) the inductive time constant of the resulting toroid? Ignore the thickness of the insulation on the wire.

## sec. 30-10 Energy Stored in a Magnetic Field

$\bullet 61$ SSM A coil is connected in series with a $10.0 \mathrm{k} \Omega$ resistor. An ideal 50.0 V battery is applied across the two devices, and the current reaches a value of 2.00 mA after 5.00 ms . (a) Find the inductance of the coil. (b) How much energy is stored in the coil at this same moment?
-62 A coil with an inductance of 2.0 H and a resistance of $10 \Omega$ is suddenly connected to an ideal battery with $\mathscr{E}=100 \mathrm{~V}$. At 0.10 s after the connection is made, what is the rate at which (a) energy is being stored in the magnetic field, (b) thermal energy is appearing in the resistance, and (c) energy is being delivered by the battery?
-63 ILW At $t=0$, a battery is connected to a series arrangement of a resistor and an inductor. If the inductive time constant is 37.0 ms , at what time is the rate at which energy is dissipated in the resistor equal to the rate at which energy is stored in the inductor's magnetic field?
-64 At $t=0$, a battery is connected to a series arrangement of a resistor and an inductor. At what multiple of the inductive time constant will the energy stored in the inductor's magnetic field be 0.500 its steady-state value?
-065 ©o For the circuit of Fig. 30-16, assume that $\mathscr{E}=10.0 \mathrm{~V}$, $R=6.70 \Omega$, and $L=5.50 \mathrm{H}$. The ideal battery is connected at time $t=0$. (a) How much energy is delivered by the battery during the first 2.00 s ? (b) How much of this energy is stored in the magnetic field of the inductor? (c) How much of this energy is dissipated in the resistor?

## sec. 30-11 Energy Density of a Magnetic Field

-66 A circular loop of wire 50 mm in radius carries a current of 100 A . Find the (a) magnetic field strength and (b) energy density at the center of the loop.
${ }^{-67}$ SSM A solenoid that is 85.0 cm long has a cross-sectional area of $17.0 \mathrm{~cm}^{2}$. There are 950 turns of wire carrying a current of 6.60 A. (a) Calculate the energy density of the magnetic field inside the solenoid. (b) Find the total energy stored in the magnetic field there (neglect end effects).
-68 A toroidal inductor with an inductance of 90.0 mH encloses a volume of $0.0200 \mathrm{~m}^{3}$. If the average energy density in the toroid is $70.0 \mathrm{~J} / \mathrm{m}^{3}$, what is the current through the inductor?
-69 ILw What must be the magnitude of a uniform electric field if it is to have the same energy density as that possessed by a 0.50 T magnetic field?
-70 ©o Figure 30-65a shows, in cross section, two wires that are straight, parallel, and very long. The ratio $i_{1} / i_{2}$ of the current carried by wire 1 to that carried by wire 2 is $1 / 3$. Wire 1 is fixed in place. Wire 2 can be moved along the positive side of the $x$ axis so as to change the magnetic energy density $u_{B}$ set up by the two currents at the origin. Figure $30-65 b$ gives $u_{B}$ as a function of the position $x$ of wire 2 . The curve has an asymptote of $u_{B}=$ $1.96 \mathrm{~nJ} / \mathrm{m}^{3}$ as $x \rightarrow \infty$, and the horizontal axis scale is set by $x_{s}=60.0$


Fig. 30-65 Problem 70. cm . What is the value of (a) $i_{1}$ and (b) $i_{2}$ ?
-•71 A length of copper wire carries a current of 10 A uniformly distributed through its cross section. Calculate the energy density of (a) the magnetic field and (b) the electric field at the surface of the wire. The wire diameter is 2.5 mm , and its resistance per unit length is $3.3 \Omega / \mathrm{km}$.

## sec. 30-12 Mutual Induction

-72 Coil 1 has $L_{1}=25 \mathrm{mH}$ and $N_{1}=100$ turns. Coil 2 has $L_{2}=40$ mH and $N_{2}=200$ turns. The coils are fixed in place; their mutual inductance $M$ is 3.0 mH . A 6.0 mA current in coil 1 is changing at the rate of $4.0 \mathrm{~A} / \mathrm{s}$. (a) What magnetic flux $\Phi_{12}$ links coil 1, and (b) what self-induced emf appears in that coil? (c) What magnetic flux $\Phi_{21}$ links coil 2 , and (d) what mutually induced emf appears in that coil?
$\bullet 73$ ssm Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate $15.0 \mathrm{~A} / \mathrm{s}$, the emf in coil 1 is 25.0 mV . (a) What is their mutual inductance? (b) When coil 2 has no current and coil 1 has a current of 3.60 A , what is the flux linkage in coil 2?
-74 Two solenoids are part of the spark coil of an automobile. When the current in one solenoid falls from 6.0 A to zero in 2.5 ms , an emf of 30 kV is induced in the other solenoid. What is the mutual inductance $M$ of the solenoids?
-075 ILw A rectangular loop of $N$ closely packed turns is positioned near a long straight wire as shown in Fig. 30-66. What is the mutual inductance $M$ for the loop-wire combination if $N=100, a=1.0 \mathrm{~cm}, b=8.0$ cm , and $l=30 \mathrm{~cm}$ ?
-076 A coil C of $N$ turns is placed around a long solenoid S of radius $R$ and $n$ turns per unit length, as in Fig. 30-67. (a) Show that the mutual inductance for the coil-solenoid combination is given by $M=\mu_{0} \pi R^{2} n N$. (b) Explain why $M$ does not depend on the shape, size, or possible lack of close packing of the coil.


Fig. 30-66 Problem 75.


Fig. 30-67 Problem 76.
©077 SSM Two coils connected as shown in Fig. 30-68 separately have inductances $L_{1}$ and $L_{2}$. Their mutual inductance is $M$. (a) Show that this combination can be replaced by a single coil of equivalent inductance given by

$$
L_{\mathrm{eq}}=L_{1}+L_{2}+2 M .
$$

(b) How could the coils in Fig. 30-68 be reconnected to yield an equivalent inductance of

$$
L_{\mathrm{eq}}=L_{1}+L_{2}-2 M ?
$$

(This problem is an extension of Problem 47, but the requirement that the coils be far apart has been removed.)


Fig. 30-68 Problem 77.

## Additional Problems

78 At time $t=0$, a 12.0 V potential difference is suddenly applied to the leads of a coil of inductance 23.0 mH and a certain re-
sistance $R$. At time $t=0.150 \mathrm{~ms}$, the current through the inductor is changing at the rate of $280 \mathrm{~A} / \mathrm{s}$. Evaluate $R$.
79 ssm In Fig. 30-69, the battery is ideal and $\mathscr{E}=10 \mathrm{~V}, R_{1}=5.0 \Omega$, $R_{2}=10 \Omega$, and $L=5.0 \mathrm{H}$. Switch S is closed at time $t=0$. Just afterwards, what are (a) $i_{1}$, (b) $i_{2}$, (c) the current $i_{\mathrm{S}}$ through the switch, (d) the potential difference $V_{2}$ across resistor 2, (e) the potential difference $V_{L}$ across the inductor, and (f) the rate of change $d i_{2} / d t$ ? A long time later, what are (g) $i_{1},(\mathrm{~h}) i_{2}$, (i) $i_{\mathrm{S}}$,


Fig. 30-69 Problem 79. (j) $V_{2}$, (k) $V_{L}$, and (1) $d i_{2} / d t$ ?

80 In Fig. $30-61, R=4.0 \mathrm{k} \Omega, L=8.0 \mu \mathrm{H}$, and the ideal battery has $\mathscr{E}=20 \mathrm{~V}$. How long after switch S is closed is the current 2.0 mA ?
81 SSM Figure $30-70 a$ shows a rectangular conducting loop of resistance $R=0.020 \Omega$, height $H=1.5$ cm , and length $D=2.5 \mathrm{~cm}$ being pulled at constant speed $v=40 \mathrm{~cm} / \mathrm{s}$ through two regions of uniform magnetic field. Figure $30-70 b$ gives the current $i$ induced in the loop as a function of the position $x$ of the right side of the loop. The vertical axis scale is set by $i_{s}=3.0 \mu \mathrm{~A}$. For example, a current equal to $i_{s}$ is induced clockwise as the loop enters region 1. What are the (a) magnitude and (b)


Fig. 30-70 Problem 81. direction (into or out of the page) of the magnetic field in region 1 ? What are the (c) magnitude and (d) direction of the magnetic field in region 2 ?
82 A uniform magnetic field $\vec{B}$ is perpendicular to the plane of a circular wire loop of radius $r$. The magnitude of the field varies with time according to $B=B_{0} e^{-t / \tau}$, where $B_{0}$ and $\tau$ are constants. Find an expression for the emf in the loop as a function of time.

83 Switch S in Fig. 30-61 is closed at time $t=0$, initiating the buildup of current in the 15.0 mH inductor and the $20.0 \Omega$ resistor. At what time is the emf across the inductor equal to the potential difference across the resistor?

84 Figure $30-71$ a shows two concentric circular regions in which uniform magnetic fields can change. Region 1, with radius $r_{1}=1.0 \mathrm{~cm}$, has an outward magnetic field $\vec{B}_{1}$ that is increasing in magnitude. Region 2, with radius $r_{2}=2.0 \mathrm{~cm}$, has an outward magnetic field $\vec{B}_{2}$ that may also be changing. Imagine that a conducting ring of radius $R$ is centered on the two regions and then the emf $\mathscr{E}$ around the ring is determined. Figure $30-71 b$ gives emf $\mathscr{E}$ as a

(b)
$R^{2}\left(\mathrm{~cm}^{2}\right)$
Fig. 30-71 Problem 84
function of the square $R^{2}$ of the ring's radius, to the outer edge of region 2. The vertical axis scale is set by $\mathscr{E}_{s}=20.0 \mathrm{nV}$. What are the rates (a) $d B_{1} / d t$ and (b) $d B_{2} / d t$ ? (c) Is the magnitude of $\vec{B}_{2}$ increasing, decreasing, or remaining constant?
85 SSM Figure 30-72 shows a uniform magnetic field $\vec{B}$ confined to a cylindrical volume of radius $R$. The magnitude of $\vec{B}$ is decreasing at a constant rate of $10 \mathrm{mT} / \mathrm{s}$. In unit-vector notation, what is the initial acceleration of an electron released at (a) point $a$ (radial distance $r=5.0 \mathrm{~cm})$, (b) point $b(r=0)$, and (c) point $c(r=5.0 \mathrm{~cm})$ ?


Fig. 30-72 Problem 85.
86 In Fig. 30-73a, switch S has been closed on $A$ long enough to establish a steady current in the inductor of inductance $L_{1}=5.00 \mathrm{mH}$ and the resistor of resistance $R_{1}=25.0 \Omega$. Similarly, in Fig. 30-73b, switch S has been closed on $A$ long enough to establish a steady current in the inductor of inductance $L_{2}=3.00 \mathrm{mH}$ and the resistor of resistance $R_{2}=30.0 \Omega$. The ratio $\Phi_{02} / \Phi_{01}$ of the magnetic flux through a turn in inductor 2 to that in inductor 1 is 1.50. At time $t=0$, the two switches are closed on $B$. At what time $t$ is the flux through a turn in the two inductors equal?


Fig. 30-73 Problem 86.

87 SSM A square wire loop 20 cm on a side, with resistance $20 \mathrm{~m} \Omega$, has its plane normal to a uniform magnetic field of magnitude $B=2.0 \mathrm{~T}$. If you pull two opposite sides of the loop away from each other, the other two sides automatically draw toward each other, reducing the area enclosed by the loop. If the area is reduced to zero in time $\Delta t=0.20 \mathrm{~s}$, what are (a) the average emf and (b) the average current induced in the loop during $\Delta t$ ?

88 A coil with 150 turns has a magnetic flux of $50.0 \mathrm{nT} \cdot \mathrm{m}^{2}$ through each turn when the current is 2.00 mA . (a) What is the inductance of the coil? What are the (b) inductance and (c) flux through each turn when the current is increased to 4.00 mA ? (d) What is the maximum emf $\mathscr{E}$ across the coil when the current through it is given by $i=(3.00 \mathrm{~mA}) \cos (377 t)$, with $t$ in seconds?

89 A coil with an inductance of 2.0 H and a resistance of $10 \Omega$ is suddenly connected to an ideal battery with $\mathscr{E}=100 \mathrm{~V}$. (a) What is the equilibrium current? (b) How much energy is stored in the magnetic field when this current exists in the coil?

90 How long would it take, following the removal of the battery, for the potential difference across the resistor in an $R L$ circuit (with $L=2.00 \mathrm{H}, R=3.00 \Omega$ ) to decay to $10.0 \%$ of its initial value?

91 SSM In the circuit of Fig. 30-74, $R_{1}=20 \mathrm{k} \Omega, R_{2}=20 \Omega, L=50 \mathrm{mH}$, and the ideal battery has $\mathscr{E}=40 \mathrm{~V}$. Switch S has been open for a long time when it is closed at time $t=0$. Just after the switch is closed, what are (a) the current $i_{\text {bat }}$ through the battery and (b) the rate $d i_{\text {bat }} / d t$ ? At $t=3.0 \mu \mathrm{~s}$, what are (c) $i_{\text {bat }}$ and (d)
$d i_{\text {bat }} / d t$ ? A long time later, what are (e) $i_{\text {bat }}$ and (f) $d i_{\text {bat }} / d t$ ?
92 The flux linkage through a certain coil of $0.75 \Omega$ resistance would be 26 mWb if there were a current of 5.5 A in it. (a) Calculate the inductance of the coil. (b) If a 6.0 V ideal battery were suddenly connected across the coil, how long would it take for the current to rise from 0 to 2.5 A ?
93 In Fig. 30-61, a 12.0 V ideal battery, a $20.0 \Omega$ resistor, and an inductor are connected by a switch at time $t=0$. At what rate is the battery transferring energy to the inductor's field at $t=1.61 \tau_{L}$ ?
94 A long cylindrical solenoid with 100 turns $/ \mathrm{cm}$ has a radius of 1.6 cm . Assume that the magnetic field it produces is parallel to its axis and is uniform in its interior. (a) What is its inductance per meter of length? (b) If the current changes at the rate of $13 \mathrm{~A} / \mathrm{s}$, what emf is induced per meter?

95 In Fig. 30-75, $R_{1}=8.0 \Omega, R_{2}=$ $10 \Omega, L_{1}=0.30 \mathrm{H}, L_{2}=0.20 \mathrm{H}$, and the ideal battery has $\mathscr{E}=6.0 \mathrm{~V}$. (a) Just after switch S is closed, at what rate is the current in inductor 1 changing? (b) When the circuit is in the steady state, what is the current in inductor 1 ?


Fig. 30-75 Problem 95.

96 A square loop of wire is held in a uniform 0.24 T magnetic field directed perpendicular to the plane of the loop. The length of each side of the square is decreasing at a constant rate of $5.0 \mathrm{~cm} / \mathrm{s}$. What emf is induced in the loop when the length is 12 cm ?
97 At time $t=0$, a 45 V potential difference is suddenly applied to the leads of a coil with inductance $L=50 \mathrm{mH}$ and resistance $R=180 \Omega$. At what rate is the current through the coil increasing at $t=1.2 \mathrm{~ms}$ ?

98 The inductance of a closely wound coil is such that an emf of 3.00 mV is induced when the current changes at the rate of 5.00 A/s. A steady current of 8.00 A produces a magnetic flux of 40.0 $\mu \mathrm{Wb}$ through each turn. (a) Calculate the inductance of the coil. (b) How many turns does the coil have?

